Quantum Quandary State Test (Division C) — Answer Key

For reference/use by graders.

Notes for graders:

- Multiple-choice answers are given by letter.
- Free-response model answers show *essential ideas*; minor wording differences are fine if the physics is correct.
- Numeric answers allow reasonable rounding. **Equivalent units** (e.g. eV vs J; nm vs m) are acceptable if the value is consistent under conversion.

I. Quantum Physics: Fundamentals & History

1. The Heisenberg uncertainty principle for position and momentum is: [*1 pt*]

Answer: (b) $\Delta x\,\Delta p \geq \frac{\hbar}{2}$.

2. Expectation value of an observable $\hat{A}$ in state $|\psi\rangle$. [*2 pts*]

Answer: $\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle$. Physically it is the average value you would obtain from many measurements of A on identically prepared systems in state $|\psi\rangle$.

3. Double-slit: if wavelength λ increases (fixed d and L), what happens to fringe spacing? [*2 pts*]

Answer: It *increases*. Using $\Delta y \simeq \frac{\lambda L}{d}$, larger λ gives larger Δy.

4. Match Pauli operators to matrices. [*2 pts*]

Answer: $\sigma_x \to$ (a), $\sigma_y \to$ (b), $\sigma_z \to$ (c).

5. Evidence for matter-wave diffraction of electrons. [*1 pt*]

Answer: (a) Davisson–Germer.

6. EPR challenged which idea? [*2 pts*]

Answer: (b) That the wavefunction provides a complete description of physical reality (they argued QM might be incomplete).

7. Orthogonality condition. [*1 pt*]

Answer: (a) $\langle \psi | \phi \rangle = 0$.

8. **Tiebreaker:** No-cloning theorem. [*3 pts*]

Answer: No-cloning: there is no physical operation that can take an arbitrary unknown state $|\psi\rangle$ and produce two copies $|\psi\rangle|\psi\rangle$ for all possible $|\psi\rangle$. A standard reason: quantum evolution is linear/unitary; if a cloner worked for two distinct inputs $|\psi\rangle$ and $|\phi\rangle$, linearity would fail for superpositions, and unitarity would also require preserving inner products in a way cloning cannot satisfy for non-orthogonal states.

9. Mach–Zehnder with $P = \cos^2(\varphi/2)$ at $\varphi = \pi/2$. [*2 pts*]

Answer: $P = \cos^2(\pi/4) = (\frac{\sqrt{2}}{2})^2 = \frac{1}{2}$. (Option (b).)

10. Why measuring in a different basis changes outcomes. [*2 pts*]

Answer: A state can be expanded in the eigenbasis of the measured observable. Separate observables generally have separate eigenvectors, so the decomposition coefficients (and thus outcome probabilities) alter when you measure in a different basis.

11. Relative phase vs global phase. [*1 pt*]

Answer: A global phase multiplies the entire state by $e^{i\theta}$ and cancels from all probabilities, while a relative phase changes phase differences between components (e.g. between $|0\rangle$ and $|1\rangle$) and can change interference/measurement probabilities in other bases.

12. Long Answer: History path to modern QM. [*8 pts*]

Answer: *Key points for full credit:*

- **Planck (1900):** Blackbody spectrum required energy quantization $E = nhf$ to avoid the ultraviolet catastrophe.
- **Einstein (1905):** Photoelectric effect explained by photons with energy hf, predicting a threshold frequency and $K_{\max} = hf - \phi$ (classical waves cannot explain the threshold/time delay features).
- **Bohr (1913):** Quantized angular momentum and discrete orbits to explain hydrogen spectral lines; predicts discrete energy levels.
- **de Broglie (1924):** Matter waves with $\lambda = h/p$ unify wave/particle behavior; supported by electron diffraction.
- **Heisenberg/Schrödinger (1925–26):** Matrix mechanics and wave mechanics provide general formalisms; Schrödinger equation gives deterministic wavefunction evolution.
- **Born/Measurement:** $|\psi|^2$ interpreted as probability density; measurement outcomes are probabilistic and depend on the observable's eigenstates.

Minor historical/wording variations are fine if the experimental puzzles and what each idea fixed are correct.

II. Atomic & Statistical Physics

13. Photoelectric: increasing intensity (fixed f above threshold) changes what most? *[1 pt]*

Answer: (c) Photocurrent (rate of emitted electrons).

14. Photoelectric stopping potential relation. *[2 pts]*

Answer: $eV_s = hf - \phi$ (equivalently $V_s = \dfrac{hf - \phi}{e}$).

15. Hydrogen transition $n = 3 \to 2$: emitted wavelength. *[2 pts]*

Answer: Energy difference:

$$\Delta E = 13.6\,\text{eV}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) = 13.6 \cdot \frac{5}{36} \approx 1.89\,\text{eV}.$$

Then $\lambda = \dfrac{hc}{\Delta E} = \dfrac{1240\,\text{eV}\cdot\text{nm}}{1.89\,\text{eV}} \approx 656\,\text{nm}$ (close to H α at 656.3 nm).

16. Compton shift at $\theta = 90^\circ$. *[2 pts]*

Answer: $\Delta\lambda = \lambda_C(1 - \cos 90^\circ) = \lambda_C(1 - 0) = \lambda_C$.

17. Tunneling probability increases when kinetic energy: *[1 pt]*

Answer: (b) Increases (closer to barrier height means less decay under the barrier).

18. Match hydrogen series to EM region. *[1 pt]*

Answer: Lyman $\to$ (a) UV; Balmer $\to$ (b) visible; Paschen $\to$ (c) IR.

19. Second law (isolated system). *[1 pt]*

Answer: The total entropy of an isolated system does not decrease: $\Delta S_{\text{total}} \geq 0$ (it increases for irreversible processes and stays constant for reversible ones).

20. Boltzmann probability and β. *[2 pts]*

Answer: $p_i = \dfrac{e^{-\beta E_i}}{Z}$ with $Z = \sum_j e^{-\beta E_j}$ and $\beta = 1/(k_B T)$.

21. When are quantum statistics important? *[2 pts]*

Answer: (b) Low T and/or high density, when thermal de Broglie wavelength becomes comparable to interparticle spacing.

22. Photon momentum for $\lambda = 400\,\text{nm}$. [*1 pt*]

Answer: $p = \dfrac{h}{\lambda} = \dfrac{6.626 \times 10^{-34}\,\text{J}\cdot\text{s}}{4.00 \times 10^{-7}\,\text{m}} \approx 1.66 \times 10^{-27}\,\text{kg}\cdot\text{m/s}$.

23. Stimulated emission and why essential for lasers. [*2 pts*]

Answer: Stimulated emission is when an incoming photon induces an excited atom to emit a second photon with the same frequency, phase, polarization, and direction (coherent copy). It is essential because it provides optical gain and coherence so light is amplified rather than just emitted randomly.

24. Two-level ratio with degeneracy. [*1 pt*]

Answer: $\dfrac{N_\epsilon}{N_0} = \dfrac{g_\epsilon}{g_0}\, e^{-\epsilon/(k_B T)}$.

25. Tiebreaker: $T = 0$ occupations for fermions vs bosons. [*3 pts*]

Answer: Ideal fermions fill all single-particle states up to the Fermi energy E_F (occupation 1 below E_F, 0 above, ignoring spin factors), forming a Fermi sea. Ideal bosons can all occupy the same lowest-energy state; at sufficiently low temperature they can macroscopically condense into the ground state (Bose–Einstein condensation), so many particles share one state rather than filling a ladder of distinct states.

26. Long answer: photoelectric vs Compton as photon evidence. [*8 pts*]

Answer: *Essential ideas:*

- **Photoelectric:** electrons are emitted only if f exceeds a threshold; $K_{\text{max}} = hf - \phi$ depends on frequency, not intensity. Intensity mainly changes the number of emitted electrons (current).
- **Compton:** scattered photon's wavelength shift depends on scattering angle: $\Delta\lambda = \lambda_C(1 - \cos\theta)$, consistent with photon momentum $p = h/\lambda$ and energy-momentum conservation in a collision.
- Classical wave theory struggles with (i) the threshold frequency and immediate emission in the photoelectric effect, and (ii) the angle-dependent wavelength shift in Compton scattering.
- Both phenomena support light behaving as quantized particles (photons) carrying energy and momentum.

III. Solid-State Materials & Quantum Instruments

27. Intrinsic semiconductor conductivity vs temperature. *[2 pts]*

Answer: (b) More carriers are thermally excited across the band gap, so conductivity increases (resistivity decreases) as T rises for intrinsic semiconductors.

28. Acceptor doping produces: *[1 pt]*

Answer: (b) p-type with hole majority carriers.

29. Intrinsic semiconductor resistivity vs temperature. *[2 pts]*

Answer: Qualitatively, resistivity decreases strongly with increasing temperature because carrier concentration rises (roughly exponentially with $-E_g/(2k_BT)$). Full credit for "resistivity decreases with T" plus a brief reason.

30. Critical magnetic field/current in a superconductor. *[2 pts]*

Answer: A critical magnetic field B_c (or critical current I_c) is the maximum field (or current) the superconductor can sustain while remaining superconducting. If exceeded, superconductivity is suppressed (transition to normal state, and resistance/flux penetration appears depending on type).

31. Proton NMR frequency at $B_0 = 3\,\mathrm{T}$. *[1 pt]*

Answer: $f = (\gamma/2\pi)B_0 \approx (42.58\,\mathrm{MHz/T})(3\,\mathrm{T}) \approx 128\,\mathrm{MHz}$. (Option (b).)

32. Why uncertainty scales as $1/\sqrt{N}$ for averaging. *[2 pts]*

Answer: For independent measurements with variance σ^2, the mean of N samples has variance σ^2/N, so the standard deviation scales as $\sigma/\sqrt{N}$. Equivalently, uncorrelated noise adds in quadrature.

33. DC Josephson relation. *[1 pt]*

Answer: (a) $I = I_c \sin\varphi$.

34. SQUID vs NV tradeoff. *[2 pts]*

Answer: Example full-credit comparison: SQUIDs can reach extremely high magnetic-field sensitivity but often require cryogenic operation and have larger sensor size; NV-diamond magnetometers can operate at/near room temperature with very high spatial resolution (nanoscale) but often have lower ultimate sensitivity and are limited by optical collection/contrast and decoherence.

35. Identify p-side, n-side, and depletion region on band diagram. [*3 pts*]

Answer: In equilibrium, the Fermi level is flat across the junction; the side where E_F lies closer to the conduction band is the n-type side, and the side where E_F lies closer to the valence band is the p-type side. The depletion region is the central band-bending region near the interface where mobile carriers are depleted (space-charge region).

36. Johnson noise scaling with bandwidth. [*2 pts*]

Answer: Doubling Δf increases V_{rms} by a factor of $\sqrt{2}$ because $V_{\text{rms}} \propto \sqrt{\Delta f}$.

37. **Tiebreaker:** Vortices in type-II superconductors. [*3 pts*]

Answer: A vortex is a localized region where superconductivity is suppressed in the core and a quantized amount of magnetic flux threads the superconductor, surrounded by circulating supercurrents. Consequences: flux penetration occurs above H_{c1}; vortices can move and cause dissipation unless pinned, affecting device noise and stability (and enabling imaging of flux patterns / limiting superconductor performance under fields).

38. Long answer: p–n junction physics and one device + noise source. [*8 pts*]

Answer: *Key points:*

- Diffusion of electrons/holes across the junction leaves behind ionized donors/acceptors, forming a depletion region with fixed charge.
- The space charge creates an internal electric field and a built-in potential that opposes further diffusion; at equilibrium drift balances diffusion.
- Band bending reflects this built-in potential; the Fermi level is flat at equilibrium.
- Device example:
  - **Diode:** forward bias lowers barrier and increases carrier injection/current; reverse bias widens depletion and suppresses current.
  - **LED:** recombination across a direct band gap emits photons with energy roughly set by E_g.
  - **Photodiode/solar cell:** illumination generates electron-hole pairs; built-in field separates charges, producing photocurrent/voltage.
- One realistic limitation/noise: Johnson noise in load resistance, shot noise from discrete charge carriers in current, $1/f$ noise from traps/defects, dark current limiting sensitivity, or finite quantum efficiency.

IV. Quantum Information Processing

39. $H|1\rangle$ equals: [*1 pt*]

Answer: $\frac{|0\rangle - |1\rangle}{\sqrt{2}}$. (Option (b).)

40. Classical mixture vs quantum superposition (interference). [*2 pts*]

Answer: A classical mixture assigns probabilities to being in $|0\rangle$ or $|1\rangle$ and has no relative phase, so it cannot show interference. A superposition $\alpha|0\rangle + \beta|1\rangle$ has a well-defined relative phase between amplitudes that can produce constructive/destructive interference when measured in another basis.

41. CNOT on input $|01\rangle$ (control = first qubit). [*1 pt*]

Answer: Outputs $|01\rangle$ (control is 0 so target is unchanged). (Option (a).)

42. Pauli-X matrix and action. [*2 pts*]

Answer: $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. It flips computational basis states: $X|0\rangle = |1\rangle$ and $X|1\rangle = |0\rangle$.

43. Bell state collapse in X basis. [*1 pt*]

Answer: (a) $|+\rangle$. For $|\Phi^+\rangle$, outcomes are perfectly correlated in the same basis, including the X basis.

44. Grover iterations for $N = 16$. [*2 pts*]

Answer: Order $\sqrt{N} = 4$ iterations; more precisely about $(\pi/4)\sqrt{16} = \pi \approx 3$ iterations. Full credit for 3 (or 4 with explanation of order-of-magnitude).

45. Why use ancilla $|-\rangle$ in Deutsch–Jozsa? [*1 pt*]

Answer: (b) It enables phase kickback so the oracle's output becomes a relative phase $(-1)^{f(x)}$ on the input register.

46. X-basis measurement dependence on relative phase φ. [*2 pts*]

Answer: Writing probabilities: $P(+) = |\langle +|\psi\rangle|^2 = \frac{1}{2}\left|\frac{1+e^{i\varphi}}{\sqrt{2}}\right|^2 = \frac{1+\cos\varphi}{2}$ and $P(-) = \frac{1-\cos\varphi}{2}$. Thus the relative phase changes interference in the X basis.

47. Why closed-system gates are reversible / unitary. [*2 pts*]

Answer: Closed-system evolution is unitary, which preserves inner products and norm (total probability). A unitary operator has an inverse ($U^{-1} = U^{\dagger}$), so the evolution is reversible; non-reversible maps generally require coupling to an environment or measurement.

48. Pauli error mapping. [*2 pts*]

Answer: Bit-flip $\leftrightarrow X$; phase-flip $\leftrightarrow Z$; both (bit+phase) $\leftrightarrow Y$ up to a global phase ($Y = iXZ$).

49. Compute $HZH|0\rangle$. [*2 pts*]

Answer: $HZH = X$, so $HZH|0\rangle = X|0\rangle = |1\rangle$. (Option (b).)

50. Tiebreaker: Why entanglement alone isn't automatic speedup; role of interference. [*3 pts*]

Answer: Entanglement is a resource but does not by itself guarantee an algorithmic advantage: a circuit can generate entanglement yet still be efficiently classically simulable or not amplify the right outcome. Many quantum algorithms work by arranging phases so that incorrect answers interfere destructively while the correct answer(s) interfere constructively, boosting their measurement probability. The speedup comes from how the algorithm uses unitary structure/oracles to shape interference patterns, not from entanglement as a standalone property.

51. Long answer: decoherence and simple QEC for bit/phase flips. [*8 pts*]

Answer: *Key ideas:*

- **Dephasing (phase damping):** randomizes relative phase between $|0\rangle$ and $|1\rangle$ without necessarily changing populations; reduces off-diagonal density-matrix elements.
- **Amplitude damping:** energy relaxation (e.g. $|1\rangle \to |0\rangle$) changes populations and tends to remove excitations.
- **Bit-flip repetition code:** encode $|0_L\rangle = |000\rangle$, $|1_L\rangle = |111\rangle$; a single X error can be detected via parity/syndrome and corrected by majority vote.
- **Phase-flip protection:** use the same repetition idea in the X basis: apply Hadamards to convert Z errors into X errors (since $HXH = Z$ and $HZH = X$), perform the bit-flip code/syndrome, then undo Hadamards.
- **Why fault-tolerant repeated correction:** physical gates/measurements are noisy; without fault tolerance errors accumulate with circuit depth. Repeated syndrome extraction and codes above threshold can suppress logical error rates enough for long computations.

Constants Sheet (Reference Only)

Fundamental Constants

Speed of light, c	2.998×10^{8} m/s
Planck constant, h	6.626×10^{-34} J·s
Reduced Planck constant, $\hbar = h/2\pi$	1.055×10^{-34} J·s
Elementary charge, e	1.602×10^{-19} C
Electron mass, m_e	9.11×10^{-31} kg
Proton mass, m_p	1.67×10^{-27} kg
Neutron mass, m_n	1.675×10^{-27} kg
Avogadro's number, N_A	6.022×10^{23} mol^{-1}
Boltzmann constant, k_B	1.381×10^{-23} J/K
Gas constant, R	8.314 J/(mol·K)
Stefan–Boltzmann constant, σ	5.670×10^{-8} W/(m^{2}K^4)
Gravitational constant, G	6.674×10^{-11} N·m^2/kg^2

Wave/Photon Relations

Photon energy $E = hf = \dfrac{hc}{\lambda}$

Momentum (photon) $p = \dfrac{h}{\lambda}$

de Broglie wavelength $\lambda = \dfrac{h}{p}$

These constants are provided for convenience. Calculators are allowed unless otherwise stated.

Common Approximations

hc	$1240\,\mathrm{eV \cdot nm}$
$k_B T$ at 300 K	$\approx 0.0259\,\mathrm{eV}$
$\hbar$	$6.582 \times 10^{-16}\,\mathrm{eV \cdot s}$